PhD Preliminary Examination in Analysis

Department of Mathematics
New Mexico Tech

Syllabus

The PhD Preliminary Examination in Analysis is intended to determine whether a student has adequate knowledge in the general area of real and complex analysis to begin a research program in applied mathematics. The exam will be written and graded by a committee of professors in the Department of Mathematics with expertise in analysis.

The exam will consist of approximately six to eight questions. The students will be given four hours to take the exam. Notes will not be allowed in the examination. A score of 70% or better will be considered passing. The committee can set a lower passing grade depending on circumstances. Students who fail the exam will be given the chance to take the exam one more time.

Students interested in taking the exam should have taken the courses MATH 435 and MATH 471 or their equivalents. A student should prepare for the exam by studying the relevant material in several of the reference books. Review problems will be provided to the students preparing for the exam. The material covered in the exam will include the following topics:

- **Real Analysis**

  1. Preliminaries.
     Logic, set theory, Cartesian product, relations, functions, countable and uncountable sets, real numbers, supremum, infimum, completeness axiom.
  2. Elements of Point Set Topology.
     Open and closed sets, limit points, compact sets, interior of a set, boundary of a set, closure of a set, Bolzano-Weierstass theorem, Heine-Borel theorem, metric spaces.
  3. Limits and Continuity.
     Convergent sequences, subsequences, Cauchy sequences, complete metric spaces, limit of a function, continuous functions, homeomorphisms, connectedness, uniform continuity, monotonicity.
  4. Differentiation.
     Mean value theorem, intermediate value Theorem, Taylor's theorem, functions of bounded variation, total variation.
  5. Integration.
     Partitions, Riemann and Riemann-Stieltjes integral, upper and lower integrals, mean value theorem, fundamental theorem of calculus, Lebesgue's criterion for Riemann integrability.

6. Sequences and Series of Functions.

   Pointwise convergence, uniform convergence, Cauchy condition for uniform convergence, Weierstrass approximation theorem, lim sup, lim inf, geometric series, alternating series, absolute convergence, conditional convergence, Weierstrass M-test, the integral test, the ratio test, the root test, Dirichlet test, Abel test, the big O, the little o.

- **Complex Analysis**

  1. Complex Numbers and Elementary Functions.

     Elementary functions, stereographic projection, extended complex plane, limits, continuity, linear fractional transformations, cross ratio.

  2. Analytic Functions and Integration.

     Complex differentiability, analyticity, Cauchy-Riemann equations, multivalued functions, Riemann surfaces, complex integration, Cauchy's theorem, Cauchy integral formula, Liouville's theorem, Morera theorem, maximum modulus theorem.

  3. Sequences, Series and Singularities of Complex Functions.

     Complex sequences and series, Taylor series, Laurent series, singularities of complex functions, analytic continuation, monodromy theorem.

  4. Residue Calculus and Contour Integration.

     Cauchy residue theorem, evaluation of definite integrals by contour integrals, principal value integrals, integrals with branch points, winding numbers, argument principle, Rouché's theorem, fundamental theorem of algebra, open mapping theorem, reflection principle, isolated singularities, meromorphic functions, Casorati-Weierstrass Theorem.

Recommended references

1. W. Rudin, *Principles of Real Analysis*, McGraw-Hill, 1976
2. T. M. Apostol, *Mathematical Analysis*, Addison-Wesley, 1974
3. S. G. Krantz, *Real Analysis and Foundations*, CRC Press, 2004
4. L. Ahlfors: *Complex Analysis*, McGraw-Hill, 1979
5. J. Bak, D. J. Newman, *Complex Analysis*, Springer, 2002
6. M. J. Ablowitz and A. S. Fokas, *Complex Variables : Introduction and Applications*, Cambridge University Press, 2003

7. J. W. Brown and R. V. Churchill, *Complex Variables and Applications*, McGraw-Hill, 2003

Note: Many previous editions of these books available on the market will suffice as well.